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A COMPARISON OF COUPLING EFFICIENCIES FOR A STIX COIL AND AN m = 1 COIL

by Donald R. Sigman

Lewis Research Center

Cleveland, Ohio 44135

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A COMPARISON OF COUPLING EFFICIENCIES FOR A STIX COIL AND AN $m = 1$ COIL

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SUMMARY

This theoretical and experimental study compares the ion-cyclotron wave generating characteristics of a Stix coil (which generates waves with azimuthal mode number $m = 0$) with those of a coil which produces primarily $m = \pm 1$ ion-cyclotron modes. The theoretical work of J. E. Hipp, which predicted very good coupling for the $m = 1$ coil, has been extended to determine the scaling laws for plasma column radius r_p and coil wavelength λ_0. Experimentally, an $m = 1$ coil ($\lambda_0 = 32$ cm) and an $m = 0$ coil ($\lambda_0 = 40$ cm) have been used to generate ion-cyclotron waves on a beam generated plasma column with electron density $n_e = 1\times10^{12}$ centimeter^{-3}. Coupling resonances with peak efficiencies of approximately 40 to 50 percent were measured for both coils in low power (~10-kW) experiments. For equal power transfer to the plasma, the $m = 0$ coil voltage was more than a factor of two greater than that for the $m = 1$ coil.

INTRODUCTION

The use of a Stix (ref. 1) coil to couple energy to the ion-cyclotron wave (ICW) in a plasma has been studied both experimentally (ref. 2 and private communication from C. C. Swett) and theoretically (refs. 1 to 5). This work has uncovered several features regarding the efficiency of coupling energy to the ICW and to the manner in which the energy carried by the wave is distributed across the plasma column. It is well known (refs. 2 and 3) that the optimum coupling efficiency occurs at electron densities n_e in the neighborhood of $2 - 5\times10^{12}$ centimeter^{-3}. For a fixed coil geometry and current, the energy coupled to the ICW varies approximately as the plasma radius to the fourth power. Because of the azimuthal symmetry of the Stix coil, the fields which heat the ions and the power flowing in the wave are largest near the outer edge of the plasma. Experiments (private communication from C. C. Swett) have verified that energy is coupled into the plasma by the Stix coil at an efficiency greater than 70 percent and that the wave generated in the plasma column has the dispersion characteristics predicted by

the theory (ref. 3).

Assuming the ICW to vary as $e^{i(kz+m\theta-\omega t)}$, one can describe the wave generating current sheet of a Stix coil as a superposition of two traveling waves (one in the +z-direction and one in the -z-direction) each having $m = 0$. That is,

$$j^* = j_o^* \sin k_o z e^{-i\omega t} = j_o^* \left(\frac{e^{ik_o z} - e^{-ik_o z}}{2i} \right) e^{-i\omega t} \tag{1}$$

(Symbols are defined in the appendix). Hipp, Kristiansen, and Hagler (ref. 6) have calculated the coupling of energy to the plasma for current sheets with $m = 0, \pm 1$. When $m = -1$ the waves travel azimuthally in the same direction that ions gyrate in the magnetic field, likewise in the direction of electron gyration when $m = +1$. For $m = \pm 1$ the wave energy is maximum near the center of the plasma column and would thus be expected to heat the innermost ions when damped in a "magnetic beach." Hipp presents the general equations for coupling to any m number mode, but only makes calculations for a particular situation (coil radius $R = 3$ cm; coil wavelength $\lambda_o = 2\pi/k_o = 15$ cm; plasma column radius $r_p = 2.5$ cm). He finds, that over the density range 1×10^{11} centimeter^{-3} $< n_e < 1\times10^{14}$ centimeter^{-3}, the $m = -1$ current sheet couples to the ICW about an order of magnitude more power per unit current than does the $m = 0$ (i.e., the plasma resistivity R_L is larger for $m = -1$). Except at very low densities, the efficiency for coupling energy to $m = +1$ modes is much less than that for $m = 0$ or $m = -1$ modes.

The coupling of energy to the ICW in a hot plasma by a short wavelength coil has been shown (ref. 5) to be inefficient because of the occurrence of ion-cyclotron damping underneath the coil. Therefore, in this report the calculations of Hipp (who assumed $\lambda_o = 15$ cm) are extended to calculate the resistivity R_L for longer wavelengths. Also, the variation of R_L with plasma radius is determined. Experimentally, a Stix-like coil (i.e., having a standing wave current sheet in the z-direction) with primary aximuthal numbers of $m = \pm 1$ has been constructed. Both this coil and a regular Stix coil are used to couple energy to the ICW in a beam plasma source. A comparison of the coupling efficiencies and radiofrequency coil currents and voltages is made.

THEORY

Geometry and Coupling Equations

The wave generating current sheet is assumed to be two wavelengths long and the current density (i.e., current per unit length) varies as $e^{i(k_o z+m\theta-\omega t)}$ (see fig. 1).

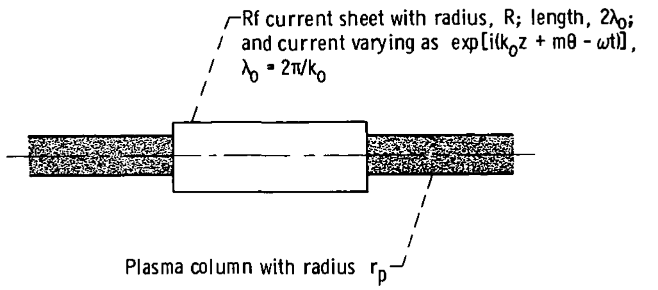


Figure 1. - Radiofrequency (rf) coil and plasma model. CS-60950

The power coupled to the ICW in a cold collisionless plasma is given by Hipp, Kristiansen, and Hagler (ref. 6) as

$$P_{av} = 2\pi i R I^2 \sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{n}{|n|} \frac{\sin^2(k_o - k_n)\lambda_o}{(k_o - k_n)^2} \frac{1}{\frac{d}{dk}\det[A]} \left\{ \det[N_7]K_m(TR)\cos\psi \right.$$

$$\left. + \left[\frac{im\gamma}{T^2}\det[N_7]K_m(TR) - \frac{-i\omega\mu_o}{T}\det[N_8]K_m(TR)\right]\sin\psi \right\}\Bigg|_{k=k_n} \tag{2}$$

The values k, for which $\det[A] = 0$, are the wave numbers k_n, of the natural modes; and the contour integral technique of Stix (ref. 1) was used to arrive at equation (2). The angle ψ represents the direction of the current relative to the z-direction ($\psi = \pi/2$ for Stix-like coils). All other terms are as defined by Hipp. The plasma resistivity is defined as

$$R_L = \frac{P_{av}}{2\pi I^2 L^2} \tag{3}$$

Numerical Calculations

Using equation (3) (with P_{av} from eq. (2)) computer calculations have been made to determine R_L for all possible combinations of three wavelengths (λ_o = 32, 40, and

48 cm), two plasma radii (r_p = 2.5 and 5.0 cm), and three densities ($n_e = 5\times10^{11}$, 5×10^{12}, and 5×10^{13} cm^{-3}). For these calculations, the current sheet radius R = 7.5 centimeters, the frequency f = 6.5 megahertz, and the pitch angle $\psi = \pi/2$.

As in all ICW calculations, there is maximum coupling when the magnetic field is such that the wavelength of the natural mode λ is close to that of the wavelength of the current sheet λ_o. Table I gives R_L with m = -1 for the various parameters discussed previously. The gas is deuterium. In each case, the magnetic field is that needed for maximum coupling. Table II gives maximum R_L with m = 0 and λ_o = 40 centimeters. The data for table II is from reference 3 and the gas in that calculation was hydrogen. The quantity ν in the last column of each table gives the variation of resistivity with plasma radius according to the empirical relation $R_L \propto r_p^\nu$ where calculated values of R_L at r_p = 2.5 and 5.0 centimeters were used to determine ν.

TABLE I. - PARAMETRIC COMPARISON OF PLASMA RESISTIVITY

R_L (m = -1 CURRENT SHEET; DEUTERIUM IONS)

Wave-length, λ_o, cm	Plasma radius, r_p, cm				ν
	2.5		5.0		
	Plasma resistivity, R_L, ohms	Nondimensional frequency, Ω[a]	Plasma resistivity, R_L, ohms	Nondimensional frequency, Ω[a]	
Electron density, n_e, 5×10^{11} cm^{-3}					
32	2.61×10^{-2}	0.989	1.72×10^{-1}	0.990	2.72
40	3.79	.983	2.32	.985	2.62
48	4.78	.977	3.04	.980	2.67
64	6.48	.959	4.24	.969	2.71
80	7.88	.946	5.30	.950	2.75
Electron density, n_e, 5×10^{12} cm^{-3}					
32	4.89×10^{-2}	0.924	2.37×10^{-1}	0.920	2.28
40	6.95	.895	3.41	.890	2.30
48	9.04	.864	4.25	.860	2.24
Electron density, n_e, 5×10^{13} cm^{-3}					
32	3.73×10^{-2}	0.73	4.90×10^{-1}	0.64	3.72
40	1.31×10^{-3}	.69	3.15	.62	7.90
48	1.26×10^{-4}	.60	9.40×10^{-2}	.60	9.51

[a] $\Omega = 2\pi f/\Omega_{ci} = \omega/\Omega_{ci}$.

TABLE II. - PARAMETRIC COMPARISON OF PLASMA RESISTIVITY

R_L (m = 0 CURRENT SHEET; HYDROGEN IONS)

Wavelength, λ_o, cm	Plasma radius, r_p, cm				ν
	2.5		5.0		
	Plasma resistivity, R_L, ohms	Nondimensional frequency, Ω[a]	Plasma resistivity, R_L, ohms	Nondimensional frequency, Ω[a]	
Electron density, n_e, 5×10^{11} cm^{-3}					
40	1.30×10^{-4}	0.95	4.13×10^{-3}	0.97	4.98
Electron density, n_e, 5×10^{12} cm^{-3}					
40	2.34×10^{-4}	0.81	5.45×10^{-3}	0.81	4.55
Electron density, n_e, 5×10^{13} cm^{-3}					
40	9.57×10^{-5}	0.43	1.95×10^{-3}	0.43	4.35

[a] $\Omega = 2\pi f/\Omega_{ci} = \omega/\Omega_{ci}$.

Discussion of Tables I and II

It is difficult to discuss the results in broad general terms. For example, the density for optimum coupling depends on both coil wavelength and plasma radius; and, although not shown in table I, on coil radius as well. One general feature is that at high densities there is a rapid falloff in R_L. The density at which this falloff begins is λ_o and r_p dependent, being lower for larger λ_o and/or smaller r_p. For example, at $\lambda_o = 40$ centimeters and $r_p = 2.5$ centimeters, R_L is reduced by a factor of 53 in going from $n_e = 5\times10^{12}$ to 5×10^{13} centimeter^{-3}. At $\lambda_o = 40$ centimeters and $r_p = 5.0$ centimeters, this factor is only 1.1. And, finally at $\lambda_o = 32$ centimeters and $r_p = 5.0$ centimeters, R_L is increased by a factor of 2.1. In Hipp's calculations ($\lambda_o = 15$ cm, $r_p = 2.5$ cm, R = 3.0 cm), the optimum density is above 10^{13} centimeter^{-3} and the rapid falloff occurs at $n_e > 10^{14}$ centimeter^{-3}. Thus, to avoid low values of R_L at high density, one must use a short wavelength coil.

Near conditions of good coupling ($n_e = 5\times10^{12}$ cm^{-3} and $r_p = 2.5$ or 5.0 cm) the following features are noted: (1) Coupling improves with increasing λ_o. However, before the optimum is reached, the coil length $2\lambda_o$ becomes too long to be practical (i.e., $\approx$200 cm). (2) Coupling improves exponentially with increasing plasma radius with the mean value of exponent ν being 2.3 (i.e., $R_L \sim r_p^{\nu}$). This is compared to $\nu = 4.6$ for

the $m = 0$ coil at $n_e = 5{\times}10^{12}$ centimeter^{-3}. It should be noted that these values of ν are only meaningful near the optimum density conditions. Thus, in designing an experiment, one must be careful not to extrapolate the data of tables I and II too far.

EXPERIMENT

Apparatus

Figure 2 is a schematic of the experimental setup. The conf. g magnetic field at the midplane was variable up to 0.45 T and the mirror ratio was 2 to 1. The distance

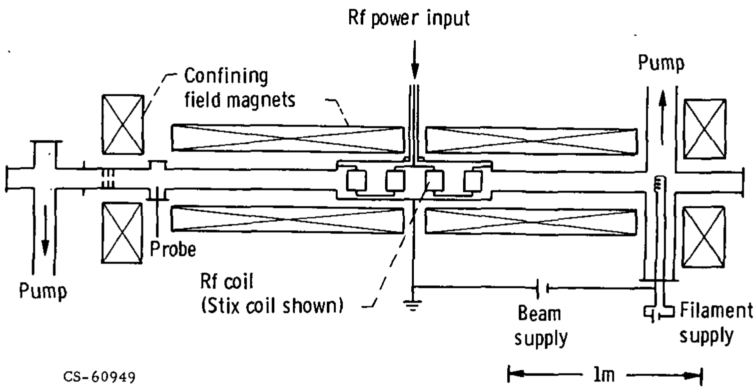
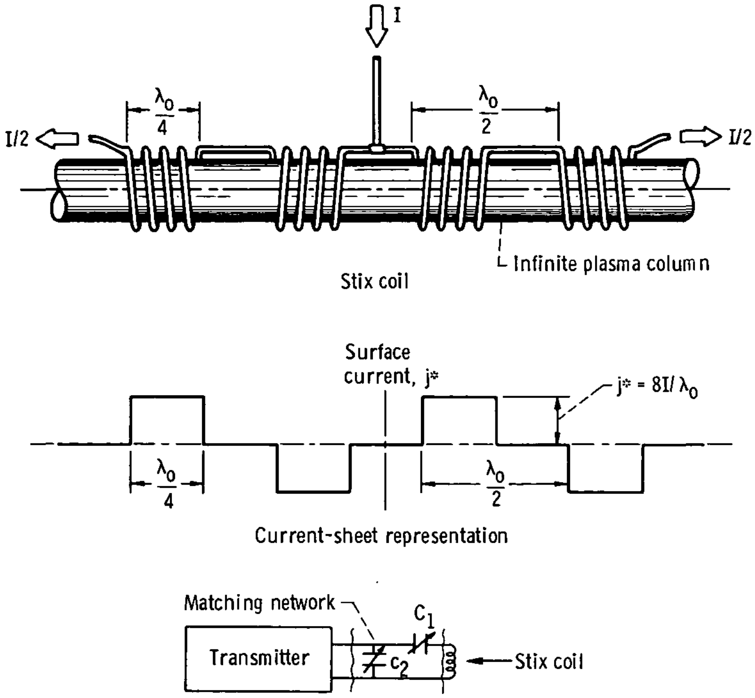


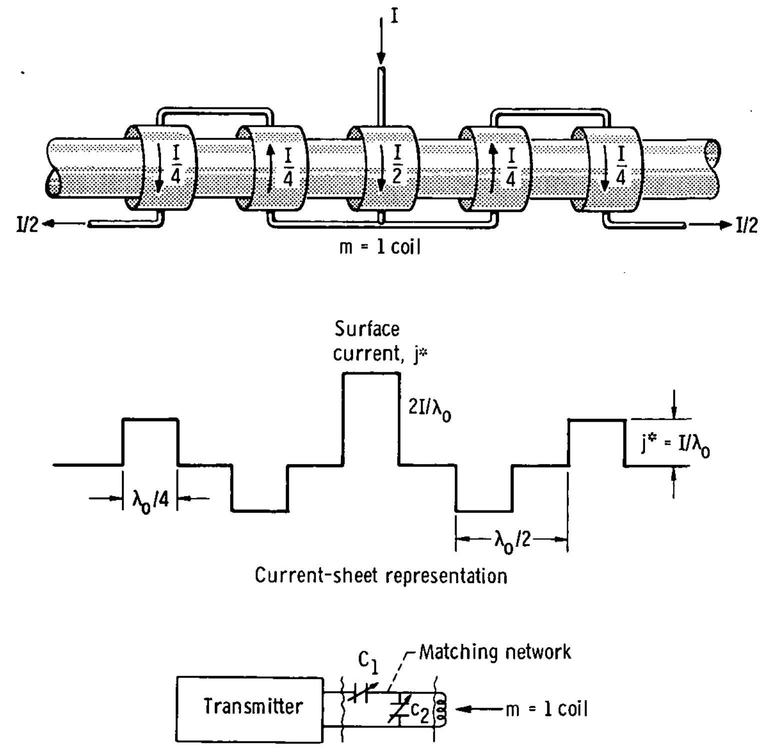
Figure 2. - Experimental arrangement for comparison of rf coils.

between mirrors was 3.0 meters and the field was constant over the plasma volume to within ±1 percent for an axial distance of ±1.2 meters from the midplane. A dc beam plasma source was used which produced a plasma of radius $r_p \approx 4$ centimeters and electron density $n_e \approx 1{\times}10^{12}$ centimeter^{-3} (as determined by a movable Langmuir probe). When the beam current was set at 100 amperes, the beam voltage was 95 volts. The typical background hydrogen gas pressure (no plasma) was $2.3{\times}10^{-3}$ torr.

The radiofrequency coil (either an $m = 1$ or $m = 0$) was excited by a 14-kilowatt transmitter with an output frequency of 6.0 megahertz. Figure 3(a) shows the two-wavelength Stix coil which was used and the circuitry used to match the input impedance of the coil to the output impedance of the transmitter (50 ohms). The coil radius $R = 7.5$ centimeters and the wavelength $\lambda_0 = 40$ centimeters. Figure 3(b) shows the $m = 1$ coil which was used and also its associated matching newwork. Again the coil radius was 7.5 centimeters. Because it was easiest to



(a) Conventional stix coil.



(b) New m = 1 coil.

Figure 3. - RF coils and matching networks used in experiment.

construct the $m = 1$ coil as a $2\frac{1}{2}$ wavelength coil, it was necessary to shorten the wavelength to 32 centimeters to make it fit the existing test section. The current distribution for this latter coil is a standing wave azimuthally. Therefore it represents a superposition of two traveling waves, one with $m = -1$ (which should be efficient in coupling energy to the ICW) and one with $m = +1$ (which should not be efficient). A grounded Faraday strip shield was used with both coils.

The inductance of the $m = 1$ coil was much less than that of the Stix coil (0.364 and 2.54 μH, respectively). However, it was found that the same variable matching capacitors C_1 and C_2 could be used by simply moving one lead.

Figure 4 shows typical measured resonance curves for the Stix coil and the $m = 1$ coil, respectively. The data were always taken under exact impedance matching conditions and always after a sufficient time to reach thermal equilibrium in the system (30 to 60 sec). Resonances measured soon after the system had been open to the atmosphere were usually poor (see solid curve in fig. 4(a)). However, after several high power runs

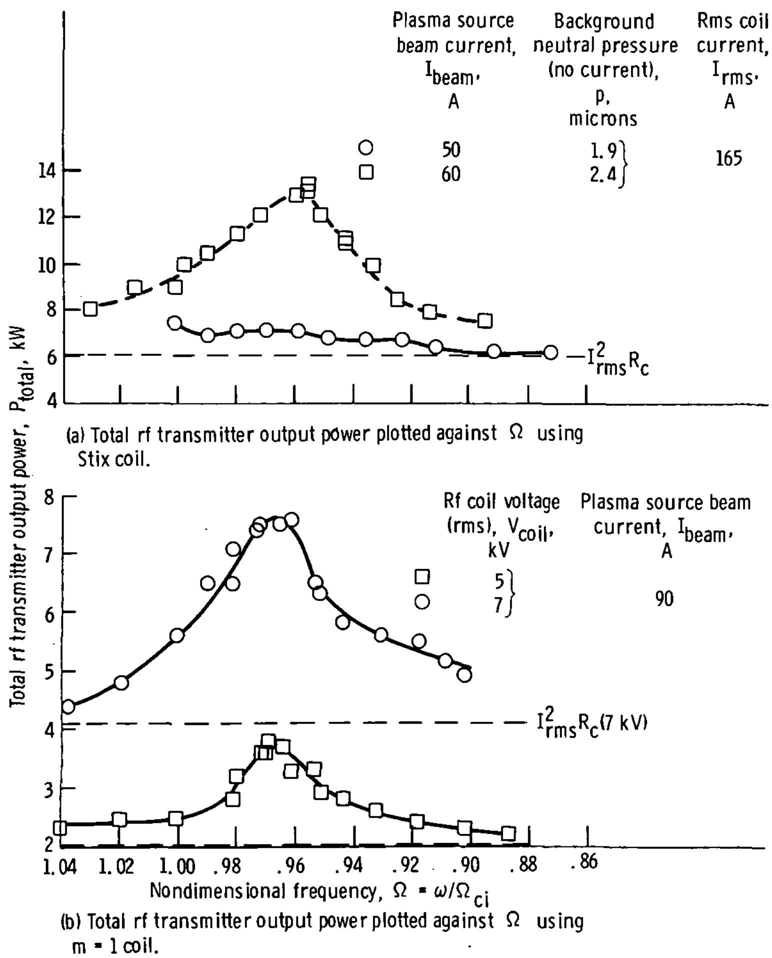


Figure 4. - Resonance absorption curves.

were made, the data resembled the remaining curves in figure 4 and did not change from run to run or even from day to day. The value of Ω for maximum coupling agrees well with theory. At $n_e = 1\times10^{12}$ centimeter^{-3}, theory predicts the maximum (for the λ_o = 40 cm Stix coil) to be at $\Omega = 0.962$. The measured value was 0.958. For the λ_o = 32 centimeters and m = 1 coil, the theoretical and experimental values are 0.974 and 0.968, respectively. These slight discrepancies might be just experimental error or they might be due to the fact that coupling radiofrequency power to the plasma increases the density thus driving the resonance peaks to lower values of Ω.

The coupling efficiency CE for power transfer to the plasma is given by

$$CE = \frac{P_{in}}{P_{in} + P_{loss}} \tag{4}$$

where P_{in} is the power transferred to the plasma and P_{loss} represents the circuit losses. Experimentally, P_{loss} is determined by running with no plasma; and P_{total} ($= P_{in} + P_{loss}$) is the net forward power when plasma is present. Table III is a summary of the resonant coupling efficiencies achieved with both coils at a variety of operating levels.

Comparison of Stix Coil and m = 1 Coil Results

Using the data of table III, we can make a comparison of the two coils by calculating the coil currents and voltages necessary to make $P_{in} = P_{total} - P_{loss} = 6$ kilowatts.

TABLE III. - SUMMARY OF m = 1 COIL

EXPERIMENTAL DATA

Coil	Peak coil voltage, V_{peak}, kV	Rf circuit power loss, P_{loss}, kW	Total of transmitter output power P_{total}, kW	Coupling efficiency, CE
m = 1	5	2.0	3.7	46.0
m = 1	7	4.2	7.6	44.8
m = 1	8	5.8	9.9	41.4
m = 1	9	7.8	13.2	40.9
Stix	22	6.3	13.0	51.5

For the Stix coil, we get V_{peak} = 20.8 kilovolts and I_{rms} = 154 amperes; and for the $m = 1$ coil, V_{peak} = 9.5 kilovolts and I_{rms} = 490 amperes. It is interesting to calculate from the aforementioned the current per unit length in each coil,

$$j^* = \frac{NI_{rms}}{\frac{\lambda_o}{2}} \tag{5}$$

where N is the number of turns per section ($N_{Stix} = 4$, $N_{m=1} = 1$). We get j^*_{Stix} = 30.8 amperes per centimeter and $j^*_{m=1}$ = 15.3 amperes per centimeter. Note that for the $m = 1$ coil the proper current to use in computing j^* is 490 amperes/2 = 245 amperes since the current arriving at each section must split into two parts.

To determine whether the values of P_{in} for $m = 0$ and $m = 1$ are related according to theory, we take a measured value of P_{in}(m = 1 coil) and apply various theoretical correction factors to calculate the expected P_{in}(Stix coil). There are five correction factors; they are the following:

(1) Correction factor for R_L: Using the available theoretical data we can calculate $R_L(r_p = 4\text{ cm})$ for $m = 0$ and 1 and for current sheets varying as $e^{i(kz+m\theta)}$. At $n_e = 5\times10^{11}$ centimeter^{-3}, $R_L(m = 0)/R_L(m = -1) = 0.0153$; and at $n_e = 5\times10^{12}$ centimeter^{-3}, $R_L(m = 0)/R_L(m = -1) = 0.0149$. Thus, we will choose the correction factor at $n_e = 1\times10^{12}$ centimeter^{-3} as 0.015.

(2) Coil turns correction factor: The number of turns per section in the Stix coil is 4 and in the $m = 1$ coil is 1. Since $P_{in} \sim N^2$, the correction factor is 16.

(3) Fourier amplitude correction factor: The axial current distribution in neither coil is sinusoidal as was assumed in the theory. Thus, the relative Fourier amplitude (evaluated at $k = k_o$) has been evaluated for each coil and the ratio of these Fourier amplitudes is 1.75.

(4) Standing to traveling wave correction factor: The $m = 1$ coil does not represent a traveling wave in the azimuthal direction, but a standing wave. Thus only one-half the amplitude ($m = -1$) is effective in coupling to the ICW. Since $P_{in} \sim (j^*)^2$ the correction factor here is 4.

(5) Relative total current correction factor: Finally, we must correct for the fact that the current per turn was not the same in each coil. This correction factor is $4I^2(m = 0)/I^2(m = 1)$ and will be evaluated later. The factor of 4 comes from the fact that, at each turn in the $m = 1$ coil, the current splits two ways.

Applying all these correction factors we get

$$P_{in}(m = 0) = P_{in}(m = 1)\,\frac{R_L(m = 0)}{R_L(m = 1)}\left[\frac{N(m = 0)}{N(m = 1)}\right]^2 (\text{Fourier amplitude factor})^2$$

$$\times (\text{Standing-traveling wave factor})^2 \left[\frac{I(m = 0)}{I(m = 1)/2}\right]^2 \qquad (6)$$

From table III we take the following $m = 1$ values: $V_{peak} = 8$ kilovolts, $I_{rms} =$ 412 amperes, and $P_{in} = 4.1$ kilowatts. Now, using equation (1) we calculate the Stix coil power $P_{in}(m = 0)$ for a current of 165 amperes:

$$P_{in}(m = 0) = 4.1 \text{ kilowatts}(0.015)(4)^2(1.75)^2\left(\frac{165}{412/2}\right)^2 = 7.8 \text{ kilowatts}$$

The measured value of $P_{in}(m = 0)$ was 6.7 kilowatts. The inclusion of $m = +1$ modes in the calculation does not explain the 15 percent discrepancy. Thus we see that the theory is quite useful in predicting $m = 0$ coupling if $m = 1$ results are known, or vice versa.

CONCLUSIONS

A theoretical parametric study of ion-cyclotron wave (ICW) coupling using an $m = -1$ traveling wave coil structure shows the following features:

1. There is a density for optimum coupling and also a density above which coupling falls off rapidly. Both densities depend strongly on coil wavelength λ_o and on plasma radius r_p being higher for shorter wavelengths and/or larger plasma radii.

2. Near the optimum coupling density, the plasma loading resistivity R_L (which is proportional to coupling) varies approximately as r_p^{ν} with $\nu = 2.6$. For an $m = 0$ Stix coil $\nu \approx 4.5$.

3. Also, near optimum coupling density R_L improves with λ_o until a maximum is reached. However, the total length of the coil ($L = 2\lambda_o$) at this maximum is probably impractical ($\lambda_o = 80$ cm for $n_e = 5{\times}10^{11}\ \text{cm}^{-3}$, $r_p = 5.0$ cm).

4. For a given coil current density j_o^*, $R_L(m = -1)$ is higher than $R_L(m = 0)$ for almost all values of λ_o, r_p, and n_e. However, the practical problems of designing a coil for high current density may be more severe for the $m = -1$ coil than for the $m = 0$ Stix coil.

Experimentally, a coil which generates both $m = \pm 1$ traveling waves and an $m = 0$ Stix coil have been used separately to couple energy to the ICW. The coupling efficiencies for the $m = 1$ coil varied inversely with the power level from 41 percent to 46 percent. The coupling efficiency for the $m = 0$ Stix coil was 51 percent. An analysis of the

different geometry of the two coils has explained the lower efficiency of the $m = 1$ coil. The larger theoretical value of $R_L(m = 1)$ has been experimentally verified; but, in order to achieve higher efficiencies, a different geometrical design is needed, with more turns per section and with an azimuthally traveling current sheet ($m = -1$).

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, February 15, 1972,
764-74.

APPENDIX - SYMBOLS

A	defined in ref. 6
C_1, C_2	rf matching network capacitances
CE	coupling efficiency, eq. (4)
f	rf driving frequency, hz
I	defined in ref. 6
I_{beam}	plasma source beam current
I_{rms}	rms coil current
j^*	rf current density
j_o^*	magnitude of j^*
$K_m(TR)$	modified Bessell function of the second kind (m^{th} order)
k	ICW wave number
k_o	rf coil wave number
L	one-half rf coil length
m	azimuthal mode number
N	number of turns/section, rf coil
N_7, N_8	defined in ref. 6
n_e	electron density
P_{in}	power transferred to the ICW
P_{loss}	rf circuit power loss
P_{total}	total rf transmitter output power
p	background neutral pressure (no plasma)
R	rf coil radius
R_L	plasma resistivity, eq. (3)
r_p	plasma radius
T	defined in ref. 6
t	time
V_{coil}	rf coil voltage (rms)
V_{peak}	peak rf coil voltage

z	axial position coordinate
γ	defined in ref. 6
θ	azimuthal position coordinate
λ	ICW wavelength
λ_o	rf coil wavelength, L
μ_o	permitivity of free space
ν	defined by eq. (4)
ψ	pitch angle of rf coil current relative to magnetic axis
Ω	nondimensional frequency, ω/Ω_{ci}
Ω_{ci}	ion-cyclotron frequency, radians
ω	wave frequency, radians

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